Autumn Scheme of learning

Year 7



The White Rose Maths schemes of learning

Why small steps?

We know that if too many concepts are covered at once, it can lead to cognitive overload, so we believe it is better to follow a small steps approach to the curriculum. As a result, each block of content in our schemes of learning is broken down into small manageable steps.

It is not the intention that each small step should last a lesson – some will be a short step withing a lesson; some will take longer than a lesson. We encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some steps alongside each other if necessary.



Teaching for mastery

Our research-based schemes of learning are designed to support a mastery approach to teaching and learning and are consistent with the aims and objectives of the National Curriculum.

Putting number first

Our schemes have number at their heart. A significant amount of time is spent reinforcing number in order to build competency and ensure students can confidently access the rest of the curriculum.

Depth before breadth

Our easy-to-follow schemes support teachers to stay within the required key stage so that students acquire depth of knowledge in each topic. Opportunities to revisit previously learnt skills are built into later blocks.

Working together

Students can progress through the schemes as a whole group, encouraging those of all abilities to support each other in their learning.

Fluency, reasoning and problem solving

Our schemes develop all three key areas of the National Curriculum, giving students the knowledge and skills they need to become confident mathematicians.



The White Rose Maths schemes of learning

Concrete - Pictorial - Abstract (CPA)

Research shows that all students, when introduced to a new concept, should have the opportunity to build competency by following the CPA approach. This features throughout our schemes of learning.

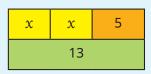
Concrete

Students should have the opportunity to work with physical objects/concrete resources, in order to bring the maths to life and to build understanding of what they are doing.



Pictorial

Alongside concrete resources, students should work with pictorial representations, making links to the concrete. Visualising a problem in this way can help students to reason and to solve problems.



Abstract

With the support of both the concrete and pictorial representations, students can develop their understanding of abstract methods.



Key Stage 3 and 4 symbols

The following symbols are used to indicate:



concrete resources might be useful to help answer the question



a bar model might be useful to help answer the question



drawing a picture might help students to answer the question



students talk about and compare their answers and reasoning



a question that should really make students think. The question may be structured differently or require a different approach from others and/or tease out common misconceptions.



the step has an explicit link to science, helping students to make cross-curricular connections.

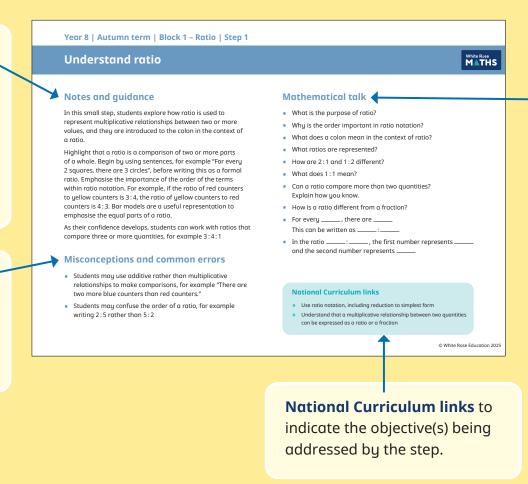
Teacher guidance

Every block in our schemes of learning is broken down into manageable small steps, with comprehensive teacher guidance for each one. Here are the features included in each step.

Notes and guidance

provide an overview of the content of the step, and ideas for teaching, along with advice on progression and where a topic fits within the curriculum.

Misconceptions and common errors are highlighted, as well as areas that may require additional support.

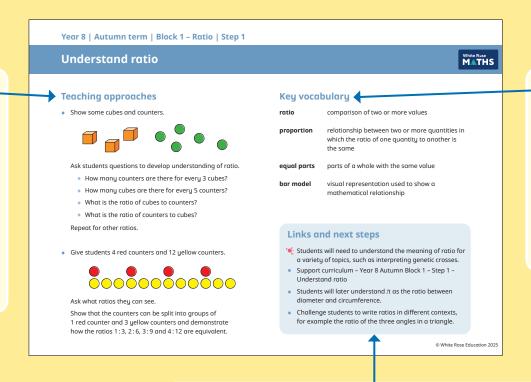


Mathematical talk

provides key questions, discussion points and possible sentence stems that can be used to develop students' mathematical vocabulary and reasoning skills, digging deeper into the content.

Teacher guidance

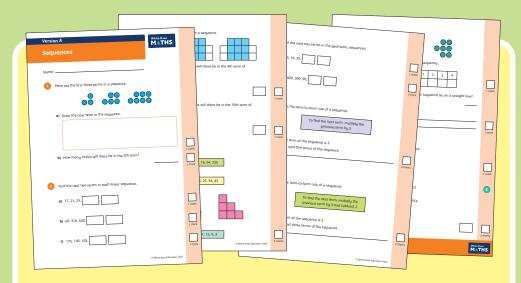
Teaching approaches — offer practical strategies for classroom use, including effective representations, modelled examples and key questions or activities designed to promote reasoning and problem solving.



Key vocabulary
emphasises the
importance of
mathematical language
and offers clear,
age-appropriate
definitions to support
understanding

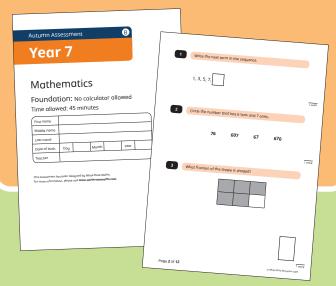
Links and next steps highlight connections to science (where appropriate) as well as alignment with the Support curriculum and shows how this step builds towards future learning. It may also include a challenge to deepen understanding, while remaining within the scope of the small step.

Free supporting materials



End-of-block assessments are provided for teachers to see how well students are progressing with the material in the curriculum. These have a total of 20 marks, assessing students' understanding of all of the steps within a block. These can be used flexibly – in the classroom, as homework, with/without a calculator, immediately after a block or later in the year – to suit teachers' and students' needs. Answers are provided.

End-of-term assessments are also provided for teachers to assess how well material is being learnt and retained in the medium and long term. There will be a calculator and non-calculator paper provided for the end of each term for each Years 7, 8 and 9. All papers will have a total of 40 marks available. We suggest 45 minutes for a paper, so that they can be done within a typical lesson. Mark schemes are provided.

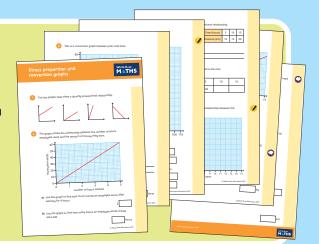


Premium supporting materials

Worksheets to

accompany every small step, providing relevant practice questions for each topic that will reinforce learning at every stage.

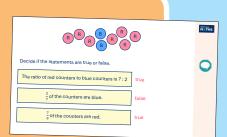
Answers to all the worksheet questions are provided.



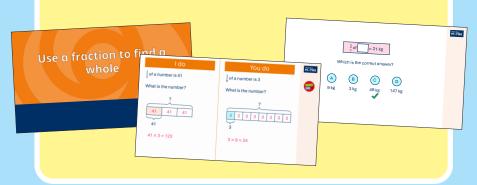
A true or false

question for every small step in the scheme of learning. These can be used to support

new learning or as another tool for revisiting knowledge at a later date.



Teaching slides for every small step, providing worked examples, multiple choice questions and open-ended questions. These are fully animated and editable, so can be adapted to the needs of any class.



1) It takes 4 people 3 hours to pave a path.
Work out how long it would take 12 people to pave the path.

1 hour

2) The number of chairs is directly proportional to the cost.
Complete the table.

| number of chairs | 1 | 2 | 3 | 4 | |
| cost (s) | 172 | |
| 3) Work out the volume of the triangular prism.

120 cm³

4 cm

4) Increase £340 by 15% £391

Flashback 4 starter activities

to improve retention.

Q1 is from the last lesson;

Q2 is from last week;

Q3 is from 2 to 3 weeks ago;

Q4 is from last term/year.

Yearly overview

The yearly overview provides suggested timings for each block of learning, which can be adapted to suit different term dates or other requirements.

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Algebra Sequences		Algebra Algebraic notation and substitution		Algebra Expressions and equations		Number Place value, ordering and rounding		Number Four operations		Statistics Averages and range	Number Rounding and estimation
Spring	Statistics Graphing data		Number Fractions, decimals and percentages		Number Directe numbe		Number Fractions and percentages of amounts		Geometry and measures Perimeter and area			
Summer	Ratio, proportion and rates of change Speed, distance and time		Number Properties of number			Number Add and subtract fractions			Geometry and measures Angles and polygons			

Autumn Block 1 Sequences

Small steps

Step 1	Describe and continue sequences
Step 2	Find the next term(s)
Step 3	Linear and non-linear sequences
Step 4	Continue linear sequences
Step 5	Continue non-linear sequences
Step 6	Term-to-term rules
Step 7	Find missing terms E

denotes an **extend step**, providing opportunities for deeper exploration of the content.



Describe and continue sequences



Notes and guidance

This small step is designed to encourage students to talk mathematically and develop their confidence in contributing to class discussion.

Students recognise and describe the change(s) from one term of a sequence to another. Ensure that they are exposed to both increasing and decreasing sequences and different types of sequences, such as linear, non-linear and oscillating. Students, however, are not expected to use these words to describe the sequences yet.

Encourage students to make the sequences using resources such as cubes or counters to help them describe and continue sequences.

Misconceptions and common errors

- Students may struggle to describe non-linear pictorial sequences when the difference between terms is not constant.
- Students may over-generalise when working with pictorial sequences, for example assuming that all sequences are linear and ascending, or that sequences cannot have negative terms.

Mathematical talk

- What is the difference between the 1st and 2nd terms?
- How could you describe how to make the next term?
- Do the terms change in the same way every time?
- To find the next term in the sequence, I need to add _____ to the previous term.
- To find the next term in the sequence, I need to remove _____ from the previous term.
- The difference between each term is _____
- Does a sequence always change from one term to the next in the same way? Explain how you know.

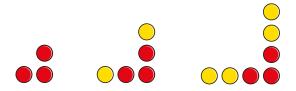
- Generate terms of a sequence from either a term-to-term or a position-to-term rule
- Recognise arithmetic sequences
- Recognise geometric sequences and appreciate other sequences that arise

Describe and continue sequences



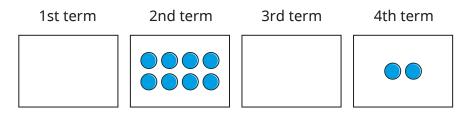
Teaching approaches

 Use different-coloured counters to create a sequence where two counters are added to the previous term.



Ask students to predict and make the 4th term of the sequence. Repeat with other terms and sequences.

- Give students some counters and ask them to make a sequence with three terms. Working in pairs, students should describe their partner's sequence and make the next term.
- Show students the 2nd and 4th terms of a sequence made from counters.



Ask students to draw the 1st and 3rd terms of the sequence. Discuss the different possible solutions.

Key vocabulary

sequence list of items in a given order, usually

following a rule

term number/object that relates to a specific

position in a sequence

position where a number or diagram is located in

a sequence

rule description to generate terms in a sequence

previous term term immediately before a given term

next term term immediately after a given term

- Students will describe patterns from observations, graphs or data to draw conclusions.
- Support curriculum Year 7 Autumn Block 1 Step 1 Sequences of diagrams
- Students will later use algebraic expressions to describe sequences.
- Challenge students to create and describe a Fibonacci sequence.

Find the next term(s)



Notes and guidance

In this small step, students identify changes in pictorial sequences and use these to work out subsequent terms.

Students count the number of objects in each term of a pictorial sequence to generate a numerical sequence. Discuss the differences between terms, asking students to predict how many objects there will be in subsequent terms. Introduce them to sequences with more than one type of object in each term, for example 1 square and 4 circles, followed by 2 squares and 6 circles, and so on, where the number of squares represents the position of each term in the sequence. Ensure that students are exposed to both linear and non-linear sequences.

Misconceptions and common errors

- When presented with sequences involving more than one type of object, students may always focus on the total number of objects in a term rather than on the types of objects.
- Students may use multiplicative reasoning to incorrectly predict terms. For example, if the 3rd term of a sequence has 7 circles, they may say that the 6th term will have 14 circles because $3 \times 2 = 6$ and $7 \times 2 = 14$

Mathematical talk

- How many objects are there in the _____ term?
 How do you know?
- Is there an efficient way to count the number of objects in each term?
 Does this help you to predict the number of objects in the 10th/100th term?
- How do the terms in the sequence change from one term to the next?
- What patterns can you see in the sequence?
- How could you check your prediction?
- The difference between each term is _____
- The _____ term will have ____ squares/circles/lines.

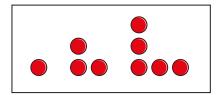
- Generate terms of a sequence from either a term-to-term or a position-to-term rule
- Recognise arithmetic sequences
- Recognise geometric sequences and appreciate other sequences that arise

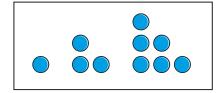
Find the next term(s)



Teaching approaches

 Show a linear and a non-linear pictorial sequence and ask what is the same and what is different.

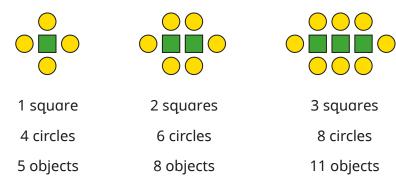




Ask students how they could work out how many circles there are in the next term and the 6th term of each sequence.

Encourage students to check their answers by drawing.

• Show a sequence made up of squares and circles.



Ask students to make predictions about other terms in the sequence. Model recording the number of squares, circles and objects in each diagram.

Key vocabulary

sequence list of items in a given order, usually

following a rule

term number/object that relates to a specific

position in a sequence

position where a number or diagram is located in

a sequence

rule description to generate terms in a sequence

term-to-term rule rule that describes how to get from one

term of a sequence to the next

- Students will be introduced to the electronic structure of elements, which follows a pictorial sequence.
- Support curriculum Year 7 Autumn Block 1 Step 2 –
 Continue number sequences
- Students will later use algebraic expressions to describe sequences and generate terms.
- Challenge students to decide if a term will occur in a sequence. For example, will there be a term in the sequence that contains 15 squares?

Linear and non-linear sequences



Notes and guidance

In this small step, students recognise the difference between linear (also known as arithmetic) and non-linear sequences and use these words to describe them.

Students learn that a linear sequence has a constant difference between terms and a non-linear sequence does not. Examples of non-linear sequences include geometric, quadratic and Fibonacci sequences. Although representing sequences graphically is not essential at this stage, appropriate technology/software could be used to highlight patterns and develop a deeper understanding of the terms "linear" and "non-linear".

Misconceptions and common errors

- Students may assume that all sequences are linear if they only look for the difference between two given terms, rather than comparing other differences in the same sequence.
- Students may assume that all non-linear sequences are geometric sequences, and therefore decide that the terms of any non-linear sequence can be generated by multiplying the previous term by a specific value.

Mathematical talk

- How is a linear sequence different from a non-linear sequence?
- How can you decide if a sequence is linear/non-linear?
- Can a linear/non-linear sequence decrease?
- The sequence has a common difference of _____
- The difference between the terms is constant, so the sequence is _____
- The difference between the terms is not constant, so the sequence is _____
- In a non-linear sequence, do the differences always increase between the terms of the sequence? Explain how you know.

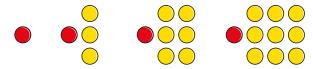
- Generate terms of a sequence from either a term-to-term or a position-to-term rule
- Recognise arithmetic sequences
- Recognise geometric sequences and appreciate other sequences that arise

Linear and non-linear sequences



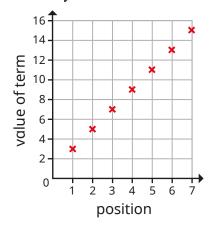
Teaching approaches

Use different-coloured counters to create a sequence.



Ask students to decide if the sequence is linear or non-linear and explain how they know. Give more examples, drawing attention to the number of counters added/subtracted each time.

Show the graph of a sequence.



Ask students questions about the graph.

- Is the sequence linear or non-linear? How do you know?
- What is the 1st/2nd/3rd term of the sequence?
- Why should the points not be joined up?

Key vocabulary

common difference constant value that is added to or subtracted from each term to get the

next term of a sequence

linear sequence sequence with a common difference

between consecutive terms

non-linear sequence sequence with no common difference

between consecutive terms

ascending sequence sequence where each term is greater

than the previous term

descending sequence sequence where each term is smaller

than the previous term

- Students will describe patterns from observations, graphs or data to draw conclusions.
- Support curriculum Year 7 Autumn Block 1 Step 5 Linear and non-linear sequences
- Students will later generate terms and algebraic expressions for linear and non-linear sequences.
- Challenge students to create a linear/non-linear sequence where the 3rd term is 10

Continue linear sequences



Notes and guidance

In this small step, students develop their understanding from earlier in the block to continue sequences. The focus is solely on linear sequences, both ascending and descending, including decimals where appropriate.

Model how to find the common difference between terms and apply this to work out subsequent terms in a given sequence. Students could also generate linear sequences from given information, for example the first term and the common difference. Calculators can be provided to offer further support and to help students develop their calculator skills.

Misconceptions and common errors

- Students may interpret the difference between terms as meaning they always "add" a value to the previous term to find the next, ignoring whether the sequence is ascending or descending.
- Students may neglect to consider inverse operations if given a term other than the 1st term and a common difference.

Mathematical talk

- How can you work out the common difference of a linear sequence?
- How does the common difference help you to work out missing terms in a linear sequence?
- How many terms do you need, to work out the common difference?
- Can a linear sequence have negative terms?
- If you know the 2nd and 3rd terms of a linear sequence, how can you work out the 1st term?
- The common difference is ______, so the next term is _____
- If you know the 1st term and the common difference of a sequence, explain why it is possible for the sequence to be completed in two different ways.

- Generate terms of a sequence from either a term-to-term or a position-to-term rule
- Recognise arithmetic sequences

Continue linear sequences



Teaching approaches

Show the first four terms of a sequence.

17, 43, 69, 95 ...

Ask questions about the sequence.

- How can you work out the common difference?
- What is the next term?
- What is the 10th term?
- What is the next term of the sequence 95, 69, 43, 17?
- Show information about two sequences.

Α

The 1st term is 5
The common difference is 2

В

The 1st term is 5
The common difference is 4

Ask students to write the first ten terms of each sequence.

Lead a class discussion on the similarities and differences between the two sequences.

Ask other questions.

- Is there more than one sequence that could be written for each set of information?
- How would the sequence change if the common difference was 8?

Key vocabulary

common difference constant value that is added to or

subtracted from each term to get the

next term of a sequence

linear sequence sequence with a common difference

between consecutive terms

ascending sequence sequence where each term is greater

than the previous term

descending sequence sequence where each term is smaller

than the previous term

- Students will describe patterns from observations, graphs or data to draw conclusions.
- Support curriculum Year 7 Autumn Block 1 Step 2 –
 Continue number sequences
- Students will later generate the terms of a linear sequence given as an algebraic expression.
- Challenge students to work out the missing terms of a fractional sequence, for example $\frac{1}{2}$, $\frac{3}{5}$, $\frac{9}{10}$, 1...

Continue non-linear sequences



Notes and guidance

In this small step, students explore non-linear sequences.

As with previous steps, encourage students to identify differences between terms to decide if the sequence is linear or non-linear.

Introduce geometric sequences by providing some terms and a rule, for example **double the previous term**. Model strategies to work out the multiplier in a geometric sequence, such as dividing the 2nd term by the 1st term. Ensure that both increasing and decreasing sequences are used throughout.

Students should also be given strategies to generate terms for other non-linear sequences, such as quadratic sequences. The use of calculators is recommended for this step to reduce cognitive load.

Misconceptions and common errors

- Students may apply methods for continuing linear sequences to non-linear sequences.
- Students may assume that all non-linear sequences are geometric and therefore have a multiplier, which is not the case for non-linear sequences such as quadratic and Fibonacci sequences.

Mathematical talk

- Do geometric sequences always increase?
- Do geometric sequences always increase/decrease faster than linear sequences?
- Is it possible for a geometric sequence to have both positive and negative terms?
- How many terms do you need to know to generate a Fibonacci sequence?
- To find the next term in the sequence, I need to multiply the previous term by _____
- To find the next term in the sequence, I need to divide the previous term by _____
- Does the difference between terms in a non-linear sequence always increase as the sequence continues? Explain how you know.

- Generate terms of a sequence from either a term-to-term or a position-to-term rule
- Recognise geometric sequences and appreciate other sequences that arise
- Recognise arithmetic sequences

Continue non-linear sequences



Teaching approaches

Show information about two sequences.

Α

A sequence is found by adding 3 to the previous term The 1st term is 2 В

A sequence is found by multiplying the previous term by 3 The 1st term is 2

Ask students to write the first five terms of each sequence.

Then ask questions about the sequences.

- What is the same/different about the sequences?
- Which sequence has a common difference?
- Which sequence is non-linear?
- Which sequence will reach 1000 first?
- Give the first four terms of a non-linear sequence that is not geometric.

2, 6, 11, 17 ...

Ask students questions about the sequence.

- Is the sequence linear or non-linear? How do you know?
- Is the sequence geometric? How do you know?
- What is the next term?
- What is the 10th term?

Key vocabulary

common difference constant value that is added to or

subtracted from each term to get the next

term of a sequence

second difference difference between the first differences

of a sequence

linear sequence sequence with a common difference

between consecutive terms

non-linear sequence sequence with no common difference

between consecutive terms

geometric sequence sequence where each successive term

is found by multiplying or dividing the previous term by the same number

- Support curriculum Year 7 Autumn Block 1 Step 5 Linear and non-linear sequences
- Students will later generate the terms of a non-linear sequence given as an iterative formula.
- Challenge students to work out the missing terms of a negative geometric sequence.

Term-to-term rules



Notes and guidance

In this small step, students find and use term-to-term rules. This step could be taught alongside the previous two steps.

Encourage students to use full sentences and mathematical vocabulary to describe the term-to-term rule for any given sequence. Remind them that linear sequences have a constant difference and ensure that they describe rules with enough detail to avoid ambiguity, for example **add 7 to the previous term**. Model strategies to work out term-to-term rules for both linear and non-linear sequences, including Fibonacci sequences. The use of calculators is encouraged to reduce cognitive load.

Misconceptions and common errors

- Students may describe a term-to-term rule with insufficient detail, for example add 3 rather than add 3 to the previous term.
- Some students may identify a non-linear sequence as linear by investigating the difference between two terms only.
- Some students may think that any sequence that can be described by a rule to get from one term to the next is a linear sequence, for example 3, 6, 12, 24 ...

Mathematical talk

- How many terms do you need, to work out a term-to-term rule?
- Is it possible to write a term-to-term rule for a sequence if the first term is unknown?
- What is the difference between an arithmetic and a geometric sequence?
- What is the same and what is different about the sequences 5, 8, 11, 14, 17 ... and 1, 4, 7, 10, 13 ... ?
- Create a linear sequence where the rule to get from one term to the next is **add 5 to the previous term**.
- How would you get from the 1st to the _____ term in this sequence?
- The common difference is _____, so the term-to-term rule is add/subtract _____ to/from the previous term.

- Generate terms of a sequence from either a term-to-term or a position-to-term rule
- Recognise arithmetic sequences
- Recognise geometric sequences and appreciate other sequences that arise

Term-to-term rules



Teaching approaches

Show the first five terms of some sequences.

Ask students to discuss what is the same and what is different about the sequences, prompting with questions.

- Which sequences are linear/non-linear?
- Are the term-to-term rules for sequences A and B the same?
- Is **add 2** a term-to-term rule?

Ask students to write the term-to-term rule for each sequence, highlighting the detail needed, for example **add 2 to the previous term**.

• Give students the first two numbers of a sequence and ask them to write a term-to-term rule. Compare the sequences generated by the students' rules and ask them to decide if the sequences are linear or non-linear.

Key vocabulary

previous term term immediately before a given term

term-to-term rule rule that describes how to get from one

term of a sequence to the next

arithmetic sequence linear sequence

geometric sequence sequence where each successive term

is found by multiplying or dividing the previous term by the same number

Fibonacci sequence sequence where the next term is found by

adding the previous two terms together

- Support curriculum Year 7 Autumn Block 1 Step 3 Term-to-term rules
- Students will apply their understanding of term-to-term rules to generate algebraic expressions to describe sequences.
- Challenge students to write a term-to-term rule for a sequence involving algebraic terms, for example 2x + y, 3x + 3y, 4x + 5y, 5x + 7y ...



Find missing terms



Notes and guidance

In this small step, students use a range of strategies to find missing terms in sequences where the rule cannot be determined from adjacent terms.

Start with finding terms that are further away than the next term in a sequence. For example, finding the 6th term given the first three terms of a sequence.

Provide examples where students are given the 1st term and another term (not the 2nd term) of a linear sequence, encouraging them to think about the number of common differences between the terms. As confidence develops, students can explore missing terms in both linear and non-linear sequences.

Misconceptions and common errors

- Students may misinterpret differences between non-consecutive terms in a sequence. For example, in the sequence 3, _____, 11 ..., students may think that the missing term is 8 because 11 3 = 8
- Some students may try to use incorrect mathematical methods to find missing terms, for example multiplying the 3rd term of a linear sequence by 2 to find the 6th term.

Mathematical talk

- How many terms are there between the 1st and 3rd terms?
- How many differences are there between the 1st and 3rd terms?
- How can you work out the 4th/5th/10th term?
- Is the sequence linear or non-linear?
- The term-to-term rule is _____
- How do your answers change, if the sequence is arithmetic?
- How do your answers change, if the sequence is geometric?
- Is it possible to have an arithmetic and geometric sequence with the same first three terms?

- Generate terms of a sequence from either a term-to-term or a position-to-term rule
- Recognise arithmetic sequences
- Recognise geometric sequences and appreciate other sequences that arise



Find missing terms



Teaching approaches

Show the first two numbers of a linear sequence.

Ask students questions about the sequence.

- What is the term-to-term rule?
- What is the 3rd term?
- What would you add to the 2nd term to find the 4th term?
- How could you work out the _____ term?
- Display linear sequences that involve procedural variation, to draw students' attention to the position of the given terms.

Ask students to work out the missing terms.

Key vocabulary

geometric sequence sequence where each successive term is found by multiplying or dividing the

previous term by the same number

previous term term immediately before a given term

linear sequence sequence with a common difference

between consecutive terms

non-linear sequence sequence with no common difference

between consecutive terms

arithmetic sequence linear sequence

term-to-term rule rule that describes how to get from one

term of a sequence to the next

position where a number or diagram is located

in a sequence

- Students will apply their understanding of term-to-term rules to generate algebraic expressions to describe sequences.
- Challenge students to find strategies to work out missing terms in a Fibonacci sequence.

Autumn Block 2 Algebraic notation

Small steps

1-step function machines (number)
1-step function machines (algebra)
Find a function (one step)
Substitution (one step)
2-step function machines (number)
2-step function machines (algebra)
Find a function (two step)
Substitution (two step)

1-step function machines (number)



Notes and guidance

In this small step, students become fluent in the use of 1-step function machines with numbers, working from both left to right and right to left.

Before finding an input from a given output, check students' understanding of inverse operations for the four operations, as well as for more complex functions such as squaring and square rooting.

Encourage the use of calculators to help students to focus on the processes. Ensure that they are confident with using a calculator for functions such as square and square root.

Misconceptions and common errors

- Students may always work from left to right with function machines, even when finding an input from a given output.
- When given functions in words, for example "subtract from 100", students may struggle to identify the inverse function.

Mathematical talk

- What is meant by "input"/"output"?
- How can you check if the answer from your calculator is sensible?
- What calculation can you do to check that your answer for the input is correct?
- What happens to the size of the output if you change the size of the input?
- If the input is _____, the output is _____
- If I know the output, I need to _____ to find the input.

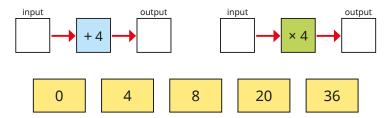
- Use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative
- Recognise and use relationships between operations including inverse operations
- Use a calculator and other technologies to calculate results accurately and then interpret them appropriately

1-step function machines (number)



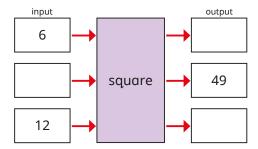
Teaching approaches

Show two 1-step function machines and some number cards.



Ask students questions to assess understanding.

- If these are the inputs, what are the outputs?
- If these are the outputs, what are the inputs?
- What is the same and what is different?
- Introduce function machines where several inputs and/or outputs are given.



Provide opportunities for students to find the input/output where the functions are written in words or symbols.

Key vocabulary

function mathematical relationship between an input

and output, written as a rule

input number, term or expression that goes into a

function and results in an output

output number, term or expression that results from

a function being applied to an input

operation mathematical process such as addition or

multiplication

inverse opposite effect of

- Support curriculum Year 7 Autumn Block 2 Step 1 –
 1-step function machines (number)
- Function machines can be used later to support substitution when solving equations.
- Challenge students to find a function, given the input and output, encouraging them to find more than one solution if possible.

1-step function machines (algebra)



Notes and guidance

In this small step, students use their knowledge of function machines from the previous step to develop an understanding of algebraic notation.

Concrete resources such as cubes and counters can be used to represent variables. Avoid using resources that have a predefined value, for example base 10, as these can lead to confusion around the definition of a variable. Discuss the conventional notation for algebra, for example 4a rather than a4 or $4 \times a$.

Misconceptions and common errors

- Students may write expressions such as $x \times 5$ as $x \times 5$ rather than 5x.
- Students may think that $\frac{x}{3}$ and $\frac{3}{x}$ are always equivalent.
- Students may write "y squared" as 2y or y2 instead of y^2
- Students may not recognise when a variable has a coefficient of 1, leading to errors such as 4b 4 = b or 4b b = 4

Mathematical talk

- What is the same and what is different about numerical and algebraic function machines?
- What does the expression 5b mean?
- Why is the expression $\frac{a}{10}$ different from $\frac{10}{a}$?
- If the input is _____, the output is _____
- ____ means ____ × ____
- Explain why 6c c is not equal to 5
- If the input is k + 3 and the function is $\times 2$, how is the output calculated?
- Is ab the same as ba? Explain how you know.

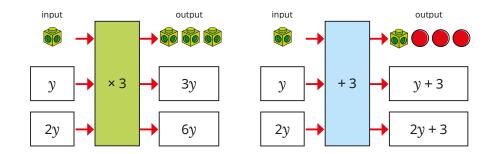
- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and $3 \times y$; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets
- Recognise and use relationships between operations including inverse operations

1-step function machines (algebra)



Teaching approaches

 Use cubes to represent variables and counters to represent ones to support understanding of algebraic notation.



Reveal the outputs one at a time and ask students questions to develop understanding.

- What is the same and what is different?
- What does 3y mean?
- Why do you add 3 counters instead of 3 cubes when adding 3?
- Investigate other machines with different functions, asking students to find the outputs for the given inputs, focusing on algebraic notation. If appropriate, include inputs with more than one term, for example 4a – 6. As their confidence develops, students can find an input from a given output.

Key vocabulary

inverse opposite effect of

variable symbol, usually a letter, that can represent any

value in mathematical expressions, identities

and formulae

term number or variable or any combination of number

and variable combined by multiplication or division

expression collection of terms involving variables and numbers

- Support Curriculum Year 7 Autumn Block 2 Step 2 –
 1-step function machines (algebra)
- Students will further develop understanding of algebraic notation when substituting later in this block and when solving equations.
- Challenge students to explore more complex functions, for example cubics or higher powers.

Find a function (one step)



Notes and guidance

In this small step, students identify the function that has taken place, given the input and the output of a function machine.

It may be useful to begin by looking at examples with just numerical inputs and outputs, before moving on to algebraic ones.

As in the previous step, using cubes and counters can help students to see the difference between multiplying a variable by a number or adding a number to a variable.

Misconceptions and common errors

- For an input of y and an output of y^2 , students may think that the function is " \times 2".
- Students may not recognise that there can be more than one possible function. For example, for an input of a and an output of a, the function could be "× 2", "+ a" or "÷ $\frac{1}{2}$ ".

Mathematical talk

- What has happened to the input to get the output?
- Is there more than one possible function?
- Will functions such as "x 2" and "+ b" always, sometimes or never give the same output from a given input?
- How many different functions can you find that will give an output of 6γ + 4 for a given input?
- Can you find at least three different functions such that the output is always the same as the input?
- If the output is less than the input, will the function always be a subtraction or a division? Explain how you know.

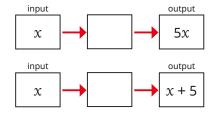
- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and $3 \times y$; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets
- Recognise and use relationships between operations including inverse operations

Find a function (one step)



Teaching approaches

Show two function machines.



Ask students what is the same and what is different about the function machines.

Model how to find the function, focusing on the operation needed to get the output from the given input. If appropriate, use cubes and counters to support students' understanding of algebraic notation.

Repeat with other examples, using a range of different variables and functions.

• Ask a student to think of a function, such as "× 2", and to stand at the front of the class, acting as the function, but without revealing the rule to the rest of the class.

Invite other students to state an input for which the student at the front gives the outputs.

Challenge the class to identify the function.

Key vocabulary

function mathematical relationship between an input

and output, written as a rule

inverse opposite effect of

variable symbol, usually a letter, that can represent any

value in mathematical expressions, identities

and formulae

term number or variable or any combination of

number and variable combined by multiplication

or division

expression collection of terms involving variables and numbers

- Support Curriculum Year 7 Autumn Block 2 Step 3 Find a function (one step)
- Challenge students to check their functions are correct by practising substitution. Students will explore substituting into expressions in more detail in the next step.

Substitution (one step)



Notes and guidance

In this small step, students substitute values into expressions, further developing their understanding of algebraic notation.

Building on the previous steps, function machines can be used in conjunction with the step if appropriate. Check that students are confident with the meaning of algebraic notation such as 2x, x^2 and $\frac{x}{2}$. This step provides an opportunity for students to develop calculator skills.

Misconceptions and common errors

- When working with an expression such as 3x, students may think that if x = 5, then 3x = 35
- For an expression such as y^2 , students may multiply the value of y by 2, rather than squaring it.
- Students may not realise that, for example, b+4 and 4+b are equivalent for any value of b.
- Students may need support with the difference between, for example, a-4 and 4-a, or $\frac{a}{4}$ and $\frac{4}{a}$
- Students may think that it is always true that a = 1, b = 2, c = 3 and so on.

Mathematical talk

- What is meant by "substitution"?
- What calculation will work out the answer?
- How can a function machine help?
- How are the expressions w 5 and 5 w different?
- Will expressions such as 2w and w^2 always, sometimes or never be equal?
- When a =______, $4a = 4 \times$ _____ = ____
- To substitute _____ into _____, I replace _____ with _____

- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and $3 \times y$; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets
- Substitute numerical values into formulae and expressions, including scientific formulae
- Recognise and use relationships between operations including inverse operations
- Use a calculator and other technologies to calculate results accurately and then interpret them appropriately

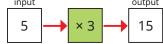
Substitution (one step)



Teaching approaches

• Tell students that a = 5 and ask them how they can use this to work out what 3a is equal to. Model using a function machine and replacing a with 5





Model the same idea using a more abstract approach, emphasising that substitution is the same as replacement.

If
$$a = 5$$
, $3a = 3 \times 5 = 15$

Repeat with other 1-step expressions, including those involving division and subtraction.

• Ask students to identify which expressions will be equivalent when p=3





$$p^2$$

Challenge students to identify which expressions will always be equivalent, whatever the value of p.

Key vocabulary

variable symbol, usually a letter, that can represent any

value in mathematical expressions, identities

and formulae

term number or variable or any combination of number

and variable combined by multiplication or

division

expression collection of terms involving variables and numbers

substitute replace letters with numerical values

- Students need to be able to substitute values into a variety of scientific formulae.
- Support Curriculum Year 7 Autumn Block 2 Step 4 –
 Substitution (one step)
- Students will revisit substitution when checking solutions to equations.
- Comparing answers of different expressions will link to sequences studied earlier and inform learning on equivalence.

2-step function machines (number)



Notes and guidance

In this small step, students move on to using two functions in a row, so that the output of the first function is the input of the second function. In this step, they become fluent with 2-step function machines with numbers, working both forwards and backwards.

Begin by checking that students are confident using inverse operations with 1-step function machines to find the input from a given output.

Encourage the use of calculators throughout if the arithmetic is a barrier.

Misconceptions and common errors

- Students may use "order of operations" to find the output, rather than completing the operations in order from left to right.
- When finding the input for a given output, students may carry out the inverse operations working from left to right.

Mathematical talk

- Does the order in which the operations are performed matter?
- How do you calculate the input when given the output?
- Why do you use inverse operations in reverse order when finding the input in a 2-step function machine?
- How can you check that your answer for the input is correct?
- How many 2-step function machines can you find that give the same output as the input?
- If the input is _____, the output is _____
- The inverse of _____ is _____

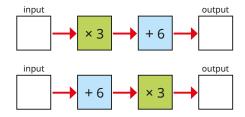
- Use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative
- Recognise and use relationships between operations including inverse operations
- Use a calculator and other technologies to calculate results accurately and then interpret them appropriately

2-step function machines (number)



Teaching approaches

Show two 2-step function machines.



Ask what is the same and what is different about the function machines.

Encourage discussion around the order in which the operations are performed and how this affects the output. Once students are confident in finding outputs, model working backwards using inverse operations to find an input for a given output, for example 21

Explore a range of "think of a number" problems.

Ask students to draw a function machine to represent the problem and then calculate the input. Encourage them to check their answer by inputting their solution back into their function machine.

I think of a number.

I multiply it by 4

I then add 5

The result is 17

What is my number?

Key vocabulary

function mathematical relationship between an input and

output, written as a rule

input number, term or expression that goes into a

function and results in an output

output number, term or expression that results from a

function being applied to an input

operation mathematical process such as addition or

multiplication

inverse opposite effect of

- Support Curriculum Year 7 Autumn Block 2 Step 5 –
 2-step function machines (number)
- Students will apply their knowledge of function machines to solve equations.
- Challenge students to find a series of functions that give an output that is equal to the input.

2-step function machines (algebra)



Notes and guidance

Building on the previous small step, students now explore 2-step function machines with algebra.

Students may need a reminder of formal algebraic notation, for example $\frac{y}{5}$ for $y \div 5$. They need to understand that the order in which the functions are applied is important. They are introduced to brackets in algebraic expressions, to distinguish between, for example, 2x + 5 and 2(x + 5). Formal expansion of brackets is not expected at this stage.

As in previous steps, cubes and counters can be used to support students' understanding.

Misconceptions and common errors

- For an input of d and functions "× 2" then "+ 5", students may give an output of 7d rather than 2d + 5
- Students may disregard the importance of order and the difference between function machines that have, for example, "+ 2" then "× 3", compared with "× 3" then "+ 2".
- When addition or subtraction is the first operation followed by multiplication or division, students may only multiply or divide the first term and disregard the use of brackets.

Mathematical talk

- How do you write $a \div 2$ using mathematical notation?
- How do you write $b \times 3$ using mathematical notation?
- How do you write c + 2 multiplied by 5 using mathematical notation?
- Is it possible that a 2-step function machine can be written as a 1-step function machine? Explain how you know.
- Does it make a difference if you change the order of a pair of functions? Explain how you know.
- 5y 3 means ____ × ___ ___
- $\frac{w}{4}$ + 1 means ____ ÷ ____ + ____

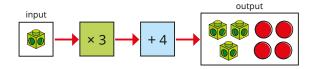
- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and $3 \times y$; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets
- Recognise and use relationships between operations including inverse operations

2-step function machines (algebra)

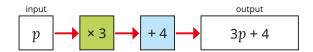


Teaching approaches

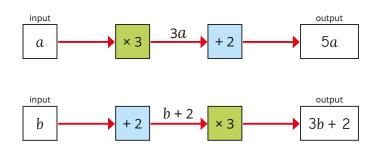
• Show a 2-step function machine with a cube as the input.



Ask students to make the output using cubes for variables and counters for ones, before revealing the answer. Show the same function machine using algebraic notation and facilitate discussion around how to write the output.



 Ask students to identify and correct the mistakes in 2-step function machines.



Key vocabulary

inverse opposite effect of

variable symbol, usually a letter, that can represent any

value in mathematical expressions, identities

and formulae

term number or variable or any combination of

number and variable combined by multiplication

or division

expression collection of terms involving variables and numbers

- Support Curriculum Year 7 Autumn Block 2 Step 6 –
 2-step function machines (algebra)
- Students will apply their knowledge of function machines to solve equations.
- Challenge students to find different 2-step function machines that give the same output as the input.

Find a function (two step)



Notes and guidance

In this small step, students develop their understanding of 2-step expressions by reversing the process of the previous step and finding the operations that formed the expressions.

Students need to recognise the order in which the operations were performed to create the expression in the output, for example recognising the difference between 6(x + 2) and 6x + 2. Reinforce that the variable represents any number.

Misconceptions and common errors

- Students may write the operations in the function machine in the wrong order.
- For expressions such as $\frac{x}{2} + 3$ and $\frac{x+3}{2}$, students may assume that division is the first operation in both cases.
- Students may fill in the missing functions with inverse operations, confusing these problems with those where they have used inverse functions to find the input given the output.

Mathematical talk

- What is the difference between $\frac{a+4}{2}$ and $\frac{a}{2}$ + 4?
- Is there more than one way of applying two consecutive functions to x and obtaining 2x + 4?
- Why are there brackets in the output?
- When is only the first term of an expression written as a fraction?
- When is an entire expression written as a fraction?
- 2x 5 means ____ × ____ ___
- $\frac{y}{3}$ + 1 means ____ ÷ ____ + ____

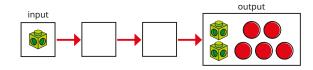
- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and $3 \times y$; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets
- Recognise and use relationships between operations including inverse operations

Find a function (two step)



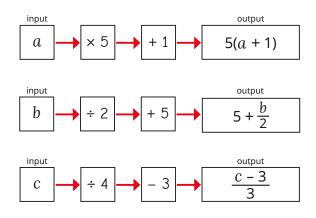
Teaching approaches

 Show students a function machine, using cubes and counters to represent the input and output.



Ask students to determine what operations have taken place and complete the functions. Repeat using algebraic notation. Encourage students to be mindful of the order in which the operations occur.

 Ask students to identify and correct any functions that are incorrect, given the inputs and outputs.



Key vocabulary

function mathematical relationship between an input and

output, written as a rule

inverse opposite effect of

variable symbol, usually a letter, that can represent any

value in mathematical expressions, identities

and formulae

term number or variable or any combination of

number and variable combined by multiplication

or division

expression collection of terms involving variables and numbers

- Understanding the order in which functions have occurred will help students with a variety of formulae used in science.
- Students will apply their knowledge of function machines to solve equations.
- Challenge students to complete the missing functions with expressions including powers and roots.

Substitution (two step)



Notes and guidance

In this small step, students substitute values into 2-step expressions, building on the skills they acquired earlier in the block.

Discuss what is the same and what is different about expressions such as 3a + 2 and 3(a + 2) to support students' understanding. Initially, students may find it useful to construct the expression as a function machine before inputting the value. As their confidence develops, they can progress to working more abstractly.

This step also allows students to practise their calculator skills, and the use of calculators is encouraged throughout.

Misconceptions and common errors

- Students may replace the unknown with the value given as a digit instead of calculating with that value. For example, when substituting x = 3 into 2x + 4, they may write this as 23 + 4 = 27
- Students may perform the calculations in the wrong order. For example, in $\frac{x+3}{2}$ they may divide the value of x by 2 first.

Mathematical talk

- When do you need to use brackets when substituting into expressions using a calculator?
- In what order should you carry out the operations?
- Can you create a function machine to represent the expression? How does this help with substitution?
- How do you square a number?
- When y = 3, $3y + 2 = 3 \times ___ + 2 = ____$
- To find the value of an expression, I need to replace ______
 with ______. Then I need to ...

- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and 3 \times y; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets
- Substitute numerical values into formulae and expressions, including scientific formulae
- Use a calculator and other technologies to calculate results accurately and then interpret them appropriately

Substitution (two step)



Teaching approaches

• Show the expression 3x + 4 alongside an empty function machine. Ask students to consider the missing operations before completing the function machine.



Now tell students that x = 2 and ask them to calculate the output using the function machine, to show that x has been replaced with the value 2



Emphasise that when x has a value of 2, the expression 3x + 4 is equal to 10

Repeat with different values of x and different expressions.

• Ask students to determine which expression is the greatest when a = 1 and b = 0.1











Repeat with different values of a and b, or challenge students to find values of a and b that make a given expression the greatest.

Key vocabulary

variable symbol, usually a letter, that can represent any

value in mathematical expressions, identities

and formulae

term number or variable or any combination of

number and variable combined by multiplication

or division

expression collection of terms involving variables and numbers

substitute replace letters with numerical values

constant term with no variable

- Students need to be able to substitute multiple values into a variety of scientific formulae, for example weight = mass × gravity
- Support Curriculum Year 7 Autumn Block 2 Step 7 –
 Substitution (two step)
- Students will revisit substitution when checking solutions to equations.

Autumn Block 3

Expressions and equations



Small steps

Step 1	Equality and equivalence
Step 2	Related facts
Step 3	Like and unlike terms
·	
Step 4	Collect like terms
Step 5	Solve 1-step equations (+/-)
,	
Step 6	Solve 1-step equations (x/÷)
Step 7	Solve 2-step equations

Equality and equivalence



Notes and guidance

In this small step, students begin to understand the meaning of equality and equivalence.

Students often misinterpret the equals sign as meaning "makes". This could become a barrier with equations such as 2 + 3 = 1 + 4. Emphasise the bidirectional nature of equality, so that students understand that the left side and right side of an equation are worth the same amount. It is helpful to read the equals sign as "is equal to" to support this.

Although collecting like terms is covered later in this block, it is important to highlight the differences between symbols used, so students should also be given opportunities to explore identities such as $3m+2m\equiv 5m$. Encourage them to substitute different values into expressions to decide if they are equivalent. Calculators may be used.

Misconceptions and common errors

- Students may apply laws of commutativity to equations involving subtraction or division.
- Students may only substitute one value into two different expressions and assume equivalence if the values of the expressions are equal.

Mathematical talk

- What difference does it make to an equation when you swap the left side and the right side?
- If you change the order of the terms on one side of an equation, will it still be correct? Explain how you know.
- What symbol is used to represent equivalence?
- Can the expressions 5 + a and 5 a ever be equal in value? Explain how you know.
- Which operations are commutative?
- How do you maintain equivalence if 1 is added to the left side of an equation?

is equal to

- Understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and 3 \times y; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets

Equality and equivalence



Teaching approaches

Show some calculations.

Ask students to identify which calculations are equal to help develop their understanding of equality.

• Show two expressions.



Ask students to choose a value for y and substitute it into each expression. Get them to repeat for different values of y, encouraging discussion about what they notice.

Highlight the fact that when y = 3, the expressions are equal, but when y is any other value, this is not the case and therefore the expressions are not equivalent.

Repeat with other pairs of expressions, such as 2w and w^2 and x + 5x and $3 \times 2x$.

Key vocabulary

equation statement to show that two expressions

are equal

commutative when an operation gives the same result

whatever the order of the terms involved

equal to the same in value

equivalent equal for all values of a given variable

- Students will build on this learning later to expand and factorise algebraic expressions.
- Challenge students to write expressions that are equivalent to a given expression, for example three expressions that are equivalent to 12y.

Related facts



Notes and guidance

In this small step, students look at calculations and expressions, using them to write related facts. They may be familiar with fact families from previous key stages, for example if 3 + 2 = 5, then 2 + 3 = 5, 5 - 2 = 3 and 5 - 3 = 2. This step extends their knowledge of fact families in preparation for solving equations and recognising equivalent forms of the same equation.

Discuss commutative and non-commutative relationships and highlight the idea of grouping versus sharing when using division. Bar models are a useful representation to highlight related facts. These can be provided and/or students can be encouraged to draw bar models from calculations or equations. They may need support drawing/interpreting bar models representing a subtraction or a division.

Misconceptions and common errors

 Students may confuse which value represents the whole and which values represent the parts, leading to incorrect related calculations.

Mathematical talk

- How many related facts is it possible to write if you know one addition fact?
- How many related facts is it possible to write if you know one subtraction fact?
- How can you show the related facts using a bar model?
- Which operations are commutative?
- Which operations are not commutative?
- Explain why y 5 is not equivalent to 5 y.
- Is it true that if x + y = a, then a x = y? Explain how you know.

- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and $3 \times y$; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; a in place of $a \times a \times b$; coefficients written as fractions rather than as decimals; brackets
- Recognise and use relationships between operations including inverse operations
- Understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors

Related facts



Teaching approaches

• Show a bar model representing the calculation 7 + 3 = 10

7	3
10	

$$7 + 3 = 10$$
 $10 - 7 = 3$

$$3+7=10$$
 $10-3=7$

Ask students what calculations they can see and write the abstract calculations beside the bar model.

• Show a bar model representing the calculation $4 \times 5 = 20$

20				
5	5	5	5	

Ask students questions to help develop understanding.

- Which calculation is equal to 5 + 5 + 5 + 5?
- What other multiplication fact can you see?
- If $20 \div 4 = 5$, what is $20 \div 5$?
- Is $20 \div 4$ the same as $4 \div 20$?

Repeat with other calculations, asking students to discuss and write the related facts.

As their confidence develops, progress to writing related facts for expressions such as 7 + y = 10 and $50 \div y = 5$

Key vocabulary

fact family related facts from a calculation or an equation

bar model visual representation used to show a

mathematical relationship

equal to the same in value

commutative when an operation gives the same result

whatever the order of the terms involved

equation statement to show that two expressions

are equal

expression collection of terms involving variables (letters)

and numbers

- Students will use their understanding from this step to solve equations.
- Challenge students to find related facts from 2-step equations such as 2x + 7 = 15

Like and unlike terms



Notes and guidance

In this small step, students identify like and unlike terms, which is vital in supporting simplification of algebraic expressions.

Students need to understand that like terms have the same variables, such as 5m and 9m, but terms with different variables, such as 3x and 3y, are unlike. Include terms with negative and fractional coefficients, so that students appreciate that, for example, 5p and -3p are like terms.

Algebra tiles can be useful to highlight the difference between linear and squared terms, for example identifying that x and x^2 are unlike terms. Discuss commutativity when comparing terms with more than one variable, for example 3ab and 8ba are like terms as $ab \equiv ba$.

Misconceptions and common errors

- Students may assume that terms with the same variable are like terms, ignoring squared terms.
- Students may think that 7a and 7b are like terms because of the coefficients or that 2mn and 5nm are not like terms because the variables are in a different order.

Mathematical talk

- Why are 3x and $3x^2$ unlike terms?
- Explain the difference between like and unlike terms.
- What is the coefficient of d in the term -d?
- Is it possible to have a pair of like terms if one is positive and one is negative? Explain how you know.
- Does the coefficient affect whether terms are like or unlike?
 Explain how you know.
- If a variable in a term is changed, does it become an unlike term? Explain how you know.
- _____ and ____ are like/unlike terms.

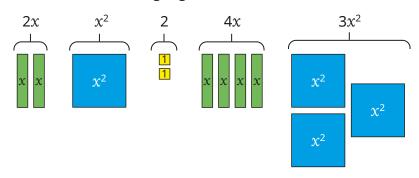
- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and 3 $\times y$; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets
- Understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors

Like and unlike terms



Teaching approaches

Show some terms using algebra tiles.



Ask students questions about the terms to help develop understanding.

- Are 2x and x^2 like terms? How do you know?
- Which term is like 2x?
- Which term is like x^2 ?

Challenge students to write other terms that are like x or x^2

 Give students some terms and ask them to sort them into sets of like terms.

3

3*a*

-3b²

-3*a*

 a^2

-3

6a

-6

6b²

6

-6*a*

12b

Key vocabulary

term number or variable or any combination

of number and variable combined by

multiplication or division

like terms terms with the same variable(s) and power(s)

unlike terms terms with different variable(s) or power(s)

coefficient number in front of a variable indicating the

multiple of the variable

expression collection of terms involving variables (letters)

and numbers

- Support curriculum Year 7 Autumn Block 3 Step 1 Like and unlike terms
- Challenge students to explore terms that include variables with a power greater than 2

Collect like terms



Notes and guidance

In this small step, students apply the ideas of equivalence to algebraic expressions.

Model simplifying expressions such as 3f + 4f by highlighting the coefficient of each term and adding these to work out the coefficient of the simplified term. Include examples of terms with a coefficient of 1 and discuss the convention of writing c rather than 1c. Work towards simplifying expressions with more than two terms and terms with squared or multiple variables, for example t^2 or 4ab, and discuss the commutative properties of like terms such as 3pq and 3qp.

Encourage students to use the \equiv symbol to indicate that expressions are identical but represented in a different way. Students should also be encouraged to identify expressions that can/cannot be simplified.

Misconceptions and common errors

- Students may misinterpret a variable with a coefficient of 1 as having no value, focusing on the numerical coefficients that are visible.
- Students may attempt to simplify expressions that do not contain like terms, for example writing $3x + 5x^2 \equiv 8x^2$

Mathematical talk

- What is the difference between equality and equivalence?
- What symbol is used to represent equivalence?
- Which of the terms are like terms?

 and are like ter 	ms
---------------------------------------	----

- _____ and ____ are unlike terms.
- What is the coefficient of _____?
- Write three different expressions that simplify to _____

$\underline{\hspace{1cm}}$ lots of x subtract $\underline{\hspace{1cm}}$	lots of	f x is	equal	tc
lots of x .				

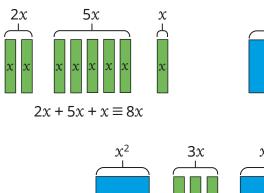
- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and $3 \times y$; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets
- Understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors
- Simplify and manipulate algebraic expressions to maintain equivalence by: collecting like terms; multiplying a single term over a bracket; taking out common factors; expanding products of two or more binomials

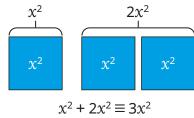
Collect like terms

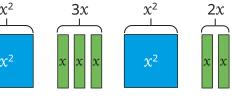


Teaching approaches

• Use algebra tiles to model how to simplify expressions.







$$x^2 + 3x + x^2 + 2x \equiv 2x^2 + 5x$$

Show two expressions with two variables.

Ask students questions about the expressions.

- What is the same? What is different?
- Are 5*ab* and 2*ba* like terms? How do you know?
- What does 5ab 2ba simplify to?

Repeat with other expressions involving addition and subtraction and more terms, as appropriate.

Key vocabulary

like terms terms with the same variable(s)

and power(s)

unlike terms terms with different variable(s)

or power(s)

equivalent equal for all values of a given variable

simplify write an expression in a simpler

equivalent form

- Support curriculum Year 7 Autumn Block 3 Step 2 –
 Collect like terms
- Students will later expand and simplify two single brackets.
- Challenge students to simplify expressions such as 3(a + b) + 2(a + b) without expanding the brackets.

Solve 1-step equations (+/-)



Notes and guidance

In this small step, students solve 1-step equations involving addition or subtraction.

Encourage students to use calculators where appropriate. Equations such as m + 5 = 9 can be useful to introduce the concept, but avoid using these too often, as students are likely to "spot" the answer rather than use inverse operations.

Refer to letters as "unknowns" rather than "variables", as for these types of equations there is only one possible solution.

Ensure that students see a variety of equations, including where the unknown is on the right-hand side of the equation, as well as variety in the order of terms, for example 293 = 35 + 3x.

Bar models may be a useful representation, building on students' understanding from previous steps. Links to function machines from the previous block could also be made.

Misconceptions and common errors

- Students may use the operation given in an equation to solve it, rather than using the inverse operation.
- Students may incorrectly think that an equation such as x 9 = 2 is the same as 9 x = 2

Mathematical talk

- What is the difference between an equation and an expression?
- What does it mean to solve an equation?
- How is an unknown different from a variable?
- What is the inverse of _____?
- How can you check if your solution is correct?
- How can you estimate what the solution will be?

- Recognise and use relationships between operations including inverse operations
- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and $3 \times y$; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets
- Understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors

Solve 1-step equations (+/-)



Teaching approaches

• Display a bar model representing the equation m + 3.8 = 5.1

5.1		
m	3.8	

Ask students to write the fact family based on the bar model.

Highlight that if m + 3.8 = 5.1, then m = 5.1 – 3.8, giving m = 1.3

Model how to check the solution of an equation by substituting it back into the original equation, explaining that both sides must have the same value.

Repeat with other equations involving addition and subtraction. Make links between the bar models and solving the abstract equation, highlighting the use of inverse operations for both.

• Display a set of equations and ask students to show that y = 60 is a solution for each one.

$$y - 36 = 24$$

$$18 + y = 78$$

$$32 = 92 - y$$

Key vocabulary

equation statement to show that two expressions are equal

inverse opposite effect of

unknown value not yet known, indicated with a letter

and used in an equation or inequality

solve find the value(s) of an unknown in an equation

solution value of an unknown that satisfies an equation

or inequality

- Support curriculum Year 7 Autumn Block 3 Step 3 –
 Solve 1-step equations (+/–)
- Students will use inverse operations to solve more complex equations throughout Key Stages 3 and 4
- Challenge students to create an equation for a given solution.

Solve 1-step equations (×/÷)



Notes and guidance

In this small step, students solve 1-step equations involving multiplication or division.

As with the previous step, calculators can be used to reduce cognitive load when finding solutions to these equations. This will allow for a variety of questions, helping to avoid misconceptions such as solutions always being integers.

Present equations with the expressions on either side of the equation. Ensure that students have the opportunity to practise 1-step equations, interchanging between multiplication and division in order to encourage them to think about which operation they need to use.

If appropriate, students can explore strategies to solve equations in the form $\frac{8}{x}$ = 5

Misconceptions and common errors

- Students may use an operation given in an equation to solve it, rather than using the inverse operation.
- Students may misinterpret an expression, for example $\frac{5}{x}$ as "x divided by 5".
- Students may think that solutions should always be integers.

Mathematical talk

- Explain why 3x = 42 is the same as 42 = 3x.
- What is the inverse of _____?
- How can you check if the solution is correct?
- To solve ______, I need to divide/multiply by _____
- What is a reasonable estimate for the solution?
- What is the same and what is different about the equations 2y = 10 and 10y = 2?
- What is the same and what is different about the equations $\frac{y}{2} = 10$ and $\frac{2}{y} = 10$?

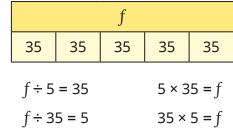
- Recognise and use relationships between operations including inverse operations
- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and 3 \times y; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets
- Understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors

Solve 1-step equations (×/÷)



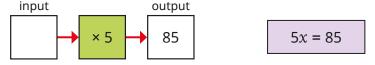
Teaching approaches

 Display a bar model and ask students to write the related fact families.



Discuss which fact(s) to use to work out the value of f. Model how to check the solution.

• Display a function machine and the related equation.



Ask students questions to develop understanding.

- What is the same? What is different?
- How could you work out the value of the input of the function machine?
- What is the value of x?

Repeat with other equations, highlighting the need for inverse operations.

Key vocabulary

inverse opposite effect of
 unknown value not yet known, indicated with a letter and used in an equation or inequality
 solve find the value(s) of an unknown in an equation value of an unknown that satisfies an equation or inequality
 coefficient number in front of a variable indicating the

multiple of the variable

- Students solve 1-step equations when working with scientific formulae such as weight = mass × gravitational field strength.
- Support curriculum Year 7 Autumn Block 3 Step 4 –
 Solve 1-step equations (×/÷)
- Challenge students to form and solve equations in context, for example area.

Solve 2-step equations



Notes and guidance

In this small step, students build on previous learning to solve 2-step equations.

As with the previous steps, emphasise the use of inverse operations to solve equations. Although "spotting" the answer may allow students to write the correct solution, this method will become gradually more difficult as equations become more complex throughout Key Stages 3 and 4

Ensure that students practise solving equations in various forms, involving both multiplication and division, rather than just those always in the form ax + b = c.

Cups and counters, bar models and function machines are useful representations to highlight key concepts. Use these alongside written abstract calculations.

Misconceptions and common errors

- Students may struggle to identify constants when equations are written in an unfamiliar form.
- Students may apply inverse operations in the wrong order when solving equations involving division, for example $\frac{t}{2} + 5 = 13$ and $\frac{t+5}{2} = 13$

Mathematical talk

- How does a bar model help you to decide what step to take first when solving a 2-step equation?
- How do you know if an equation can be solved in one step or will need more than one step?
- How can you check if the solution is correct?
- Can an equation have a non-integer solution? Explain how you know.
- Do you always need to add or subtract as a first step when solving a 2-step equation? Explain how you know.

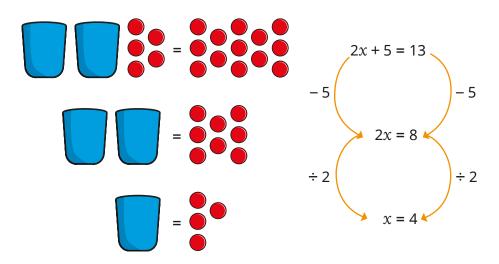
- Recognise and use relationships between operations including inverse operations
- Use and interpret algebraic notation, including: ab in place of $a \times b$; 3y in place of y + y + y and 3 \times y; a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$; a^2b in place of $a \times a \times b$; $\frac{a}{b}$ in place of $a \div b$; coefficients written as fractions rather than as decimals; brackets
- Understand and use the concepts and vocabulary of expressions, equations, inequalities, terms and factors

Solve 2-step equations



Teaching approaches

Use cups and counters to model how to solve an equation.



Make links to the abstract method and the use of inverse operations.

• Display pairs of related equations and discuss the similarities and differences in solving them.

$$3x + 15 = 63$$

$$3x - 15 = 63$$

$$\frac{y}{5} + 3 = 8$$

$$\frac{y+3}{5} = 8$$

Key vocabulary

equation statement to show that two expressions are equal

inverse opposite effect of

unknown value not yet known, indicated with a letter

and used in an equation or inequality

solve find the value(s) of an unknown in an equation

solution value of an unknown that satisfies an equation

or inequality

coefficient number in front of a variable indicating the

multiple of the variable

- Students solve 2-step equations when working with scientific formulae.
- Support curriculum Year 8 Spring Block 2 Step 3 Solve 2-step equations
- Challenge students to form and solve equations from given information, for example "think of a number" problems.

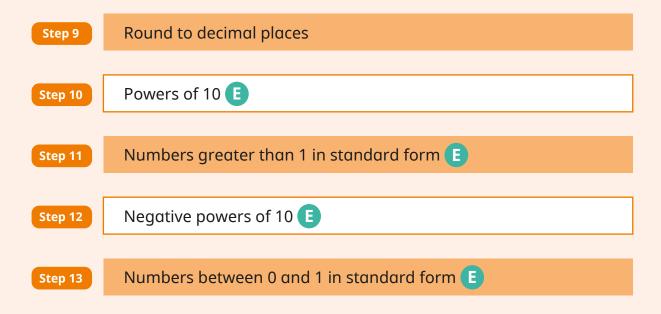
Autumn Block 4

Place value, ordering and rounding

Small steps

Write integers in numerals and words
Intervals on a number line
Compare and order integers
Place value for decimals
Decimals on a number line
Compare and order decimals
Round to powers of 10
Round to the nearest integer

Small steps



denotes an **extend step**, providing opportunities for deeper exploration of the content.

Write integers in numerals and words



Notes and guidance

In this small step, students practise reading and writing integers. Students have met numbers up to ten million at Key Stage 2, where they used commas as separators, but this scheme uses the same convention as GCSE, with no separators in numbers up to 9999 and spaces in numbers from 10 000

Start by checking students' understanding of integer place value. Place value charts and part-whole models are useful for supporting students. Encourage them to focus on the structure of an integer, for example saying 5027 as "five thousand and twenty-seven" rather than just "five-zero-two-seven". It is useful to apply a context to this step, such as population or government finances.

Misconceptions and common errors

- Students may struggle with where to position the spaces in large numbers.
- Students may not recognise large numbers written with no spaces.
- Students may find numbers with several placeholders difficult to interpret, for example 1008070

Mathematical talk

- Why are placeholders needed?
- How does the place value chart help you to represent the number?
- What is the value of each digit in the number?
- Why are spaces used in large integers?
- What strategies can you use to work out the value of a digit in a very large integer?
- How do you write "one million" in numerals and words?
- How do you write "half a million" in numerals and words?

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ic road	ac
 is read	U.S

National Curriculum links

 Understand and use place value for decimals, measures and integers of any size

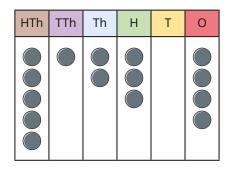
Write integers in numerals and words



Teaching approaches

Represent pairs of numbers in place value charts.

HTh	TTh	Th	Н	Т	0



Ask students to compare pairs of numbers, drawing attention to place value and placeholders. Prompt them to read the numbers aloud and to write the numbers in numerals and words.

Give seven students a digit card each.



Ask the students to stand in a line. Ask the rest of the class to say and write the number in words.

Repeat with the students in different positions and discuss how the values of the digits have changed.

Key vocabulary

numeral symbol that represents a number

digit any numeral from 0 to 9

placeholder zero that holds a place and changes the value

of other digits

place value value of a digit that relates to its position in

a number

integer whole number that can be positive or negative

- Support curriculum Year 7 Autumn Block 4 Step 1 Read and write integers to 10000
- Support curriculum Year 7 Autumn Block 4 Step 2 Understand the place value of a digit in an integer to 10000
- Students will use place value when multiplying and dividing integers by powers of 10
- Challenge students to work out calculations given integers in both numerals and words, for example finding the difference between one billion and 75 000 000

Intervals on a number line



Notes and guidance

In this small step, students work out intervals and position integers on number lines.

Start with number lines where the start and end values are given, before exploring number lines where other values are given. Once students can demonstrate how to calculate intervals, they can start to use these to read values and position integers. Use number lines with a varying number of intervals and unmarked values in different positions. Making links to reading from common scales such as rulers, weighing scales and measuring jugs can be helpful. Students can then progress to estimating the position of an integer where fewer marks are given. Encourage them to find the midpoints of marked intervals to help them estimate. If required, students can use calculators for support.

Misconceptions and common errors

- Students may count the number of marks, rather than the intervals on a number line.
- Students may incorrectly calculate a midpoint, for example thinking that the midpoint between 1000 and 5000 is 2500

Mathematical talk

- What are the values at the start and end points of the number line?
- What is the difference in value between the start and end points?
- How many intervals are there?
- What do you notice about the spacing between the intervals on the number line?
- Why is it important for intervals on a number line to be evenly spaced?
- How can you work out what each interval is worth?
- Why do you count the number of intervals rather than the number of marks on a number line?
- How can you work out the midpoint of the interval?
- Why can you mark some numbers exactly on a number line, but only estimate others?
- There are _____ intervals. Each interval is worth _____

National Curriculum links

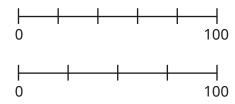
 Understand and use place value for decimals, measures and integers of any size

Intervals on a number line



Teaching approaches

• Present students with two number lines and ask them to discuss what is the same and what is different.



Ask questions to assess understanding.

- How many intervals are there between 0 and 100?
- What is the value of each interval?

Ask students to position 80 on each number line, discussing the accuracy of each placement.

Repeat with different numbers of intervals on the number lines, using appropriate end point values, for example 50, 200 or 1000

Move on to varying both the start and end point values, for example 100 to 150 or 2000 to 3000

Ask students to draw a number line that is, for example,
 20 cm long, labelling the end points 0 and 20

Ask them to estimate values and mark them on the number line, measuring to check their accuracy.

Key vocabulary

interval space between marks that splits a number line or

scale into equal parts

midpoint halfway between two intervals on a number line

or two numbers

estimate number or result that is not exact, but is suitably

close to the actual value

- Students use number lines to show the melting or boiling point of a substance or to find the state of a substance at a given temperature.
- Support curriculum Year 7 Autumn Block 4 Step 5 –
 Work out intervals on a number line
- Support curriculum Year 7 Autumn Block 4 Step 6 –
 Position integers on a number line
- Number lines will be revisited when students explore directed numbers, fractions and decimals, as well as when reading graphs and working with the probability scale.

Compare and order integers



Notes and guidance

In this small step, students compare and order integers up to one billion.

Students should use inequality symbols and precise mathematical language such as "greater than" and "less than". Encourage them to read statements such as "1067 < 1607" from both left to right and right to left. They should be familiar with the equals symbol, but may need to be introduced to the symbol for "not equal" (\neq).

Students can then use their skills of identifying the values of digits in a number, supported by number lines if necessary, to put a series of integers in ascending or descending order. Emphasise the difference between a number and a digit, as students can get confused. Using numbers in different contexts, such as masses, wages and costs, may be useful.

Misconceptions and common errors

 Students may look at the size of the leading digit and not consider the place value of the digits within the numbers, for example thinking that 9587 is greater than 10000, because 9 is greater than 1

Mathematical talk

- What do you look at first when comparing the size of two integers?
- How is comparing numbers with the same number of digits different from comparing numbers with different numbers of digits?
- Is it always, sometimes or never true that if a > b and b > c, then a > c?
- For a set of integers, is the number with the most digits always the greatest number? Explain how you know.
- What is the difference between ascending and descending order?
- Why is the leading digit of a number important when ordering?
- Two numbers have the same digits, but one is greater than the other. What could the numbers be?

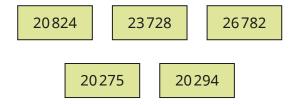
- Understand and use place value for decimals, measures and integers of any size
- Order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥

Compare and order integers



Teaching approaches

Show some 5-digit numbers.



Ask students to write the numbers in ascending/descending order. Encourage them to justify their comparisons, prompting them to use a place value chart if necessary.

Ask students how the order changes if the digits in each number are reversed. Challenge them to write a different 5-digit number that will be second greatest/smallest in the list. Repeat with fewer or greater numbers of digits.

 Give students some numbers in context, for example average salaries for different jobs, and ask them to compare and order the salaries.

Key vocabulary

compare look at quantities to find out whether their

value is greater than, equal to or less than

leading digit first digit in a number

equal to same in value

greater than more than another amount

less than smaller than another amount

ascending order ordered from smallest to greatest

descending order ordered from greatest to smallest

- Students compare and order numbers such as the distance between different planets and the Sun.
- Support curriculum Year 7 Autumn Block 4 Step 3 Compare integers to 10000
- Support curriculum Year 7 Autumn Block 4 Step 4 –
 Order integers to 10 000
- Students will later use inequalities to compare decimal and negative numbers.

Place value for decimals



Notes and guidance

In this small step, students identify the value of the digits in a decimal number and partition them using place value charts.

Discuss the difference between integer and decimal parts of numbers, for example understanding that 4.5 is 4 ones and 5 tenths. Students can also explore the relationship between the different place value columns, for example hundredths are 10 times the value of thousandths and one-tenth of the value of tenths. Encourage them to focus on the structure of decimal numbers, for example saying 1.35 as "one point three five" rather than "one point thirty-five", which can lead to misconceptions such as 1.35 > 1.4

Misconceptions and common errors

- Students may confuse the words "thousand" and "thousandth", "hundred" and "hundredth", and "ten" and "tenth".
- Students may use the incorrect number of placeholders, so write the incorrect number.

Mathematical talk

- How does the place value chart help you to represent the number?
- What is the value of each digit in the number?
- How many tenths are there in 1 whole?
- How many thousandths are there in 1 hundredth?
- What is the same and what is different about the numbers 0.7 and 0.07?
- Explain why 8.5 = 8.50
- How do you write "65 tenths" in numerals?
- Explain why 0.87 said aloud is not "nought point eighty-seven".
- What digit is in the _____ column?
- What is the value of the digit _____ in the number _____?

National Curriculum links

 Understand and use place value for decimals, measures and integers of any size

Place value for decimals



Teaching approaches

Use a place value chart to explore reading decimal numbers.

Ones	Tenths	Hundredths
•		

Ask students to explain why the number shown is 2.15, drawing attention to the place value of each digit.

Develop students' understanding of the number 2.15 in words, for example 2 ones, 1 tenth and 5 hundredths, as well as partitioning numerically, for example 2.15 = 2 + 0.1 + 0.05

Repeat for numbers with a different number of decimal places and/or placeholders, for example 0.34, 3.4 and 3.004

 Give some numbers in context, such as the masses of some dogs.

8.61 kg 6.4 kg 19.03 kg 30.27 kg

Ask students to identify the place value of each digit.

Key vocabulary

decimal number number with a whole and a fractional part,

for example 3.2, 5.43

place value value of a digit that relates to its position in

a number

tenth 1 part out of 10 equal parts

hundredth 1 part out of 100 equal parts

thousandth 1 part out of 1000 equal parts

decimal place position of a digit to the right of the decimal point

- Students read and write decimals when using a digital scale, such as a mass balance (weighing scale).
- Support curriculum Year 7 Autumn Block 4 Step 7 Place value for decimals
- Students will later convert between fractional and decimal forms of tenths, hundredths and thousandths.
- Students will use their knowledge of place value to complete calculations involving decimal numbers.

Decimals on a number line



Notes and guidance

In this small step, students focus on understanding how the place value of decimal numbers affects their relative positioning on a number line. They should already be confident with integer number lines and be aware of the difference between the number of marks and the number of intervals.

Start with number lines where the start and end values are given, focusing on key decimals such as 0.5, 0.1, 0.2 and 0.25. As students' confidence develops, use number lines that have a varying number of intervals and unmarked values in different positions. Students can then progress to finding the midpoints of intervals and intervals such as 0.05 and 0.001, if appropriate.

Misconceptions and common errors

- Students may assume that each interval is 0.1 without checking other numbers on the number line.
- Students may incorrectly calculate a midpoint of two decimal numbers, for example thinking that the midpoint of 5.6 and 5.7 is 5.61

Mathematical talk

- Why do you count the number of intervals rather than the number of marks on a number line?
- How do you work out the size of an interval on a number line?
- What is different when thinking about the positions of 0.3 and 0.03?
- How many intervals are there between each number?
- If there are 10 equal intervals between two consecutive whole numbers, what is each interval worth?
- Where would you place 0.5 on a number line between 0 and 1?
 Explain your answer.
- There are _____ intervals. Each interval is worth _____

National Curriculum links

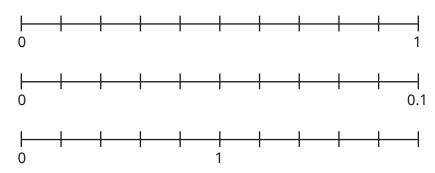
 Understand and use place value for decimals, measures and integers of any size

Decimals on a number line



Teaching approaches

Show three number lines.



Ask students questions to develop understanding.

- What is the same and what is different?
- How many intervals are there between the given numbers?
- What is the value of each interval?

Ask students to label the value of each mark.

Repeat with a different number of intervals, varying both the start and end point values.

 Ask two students to stand at either side of the classroom, holding digit cards showing 0 and 1, creating a "number line".
 Ask other students to position themselves on the "line" to show the approximate positions of 0.5, 0.25 and so on.

Key vocabulary

interval space between marks that splits a number line

or scale into equal parts

decimal number number with a whole and a fractional part,

for example 3.2, 5.43

decimal point dot to the right of the ones column that is used

to separate the integer and the decimal parts

tenth 1 part out of 10 equal parts

hundredth 1 part out of 100 equal parts

thousandth 1 part out of 1000 equal parts

midpoint halfway between two intervals on a number

line or two numbers

Links and next steps

 The number line is a key representation to develop fluency and confidence with equivalent decimal, fractional and percentage values, as well as reading graphs and working with the probability scale.

Compare and order decimals



Notes and guidance

In this small step, students compare and order decimal numbers.

Start with decimal numbers that have the same number of decimal places, before progressing to other decimals and/or integers. It is important that students consider the values of the digits in place value order. Discuss whether all the place value columns need to be compared. For example, when comparing 4.62 and 9.18, only the ones need to be compared, but when comparing 0.294 and 0.29, all the places need to be compared. Representations such as place value charts and number lines can be used for support. Using numbers in context, such as lengths, costs and masses, may be useful.

Misconceptions and common errors

- Students may not realise that, for example, 0.2 = 0.20
- Students may think that 0.65 is greater than 0.8 because 65 is greater than 8
- Students may compare 0.4, 0.34 and 0.403 as though they are comparing 4, 34 and 403 and ignore the place values of the digits.

Mathematical talk

- When ordering numbers, why are the leading digits important?
- Do you need to look at every place value column when ordering these numbers? Explain your answer.
- How many tens/ones/tenths/hundredths does the number have?
- Why do you say 4.89 as "four point eight nine" rather than "four point eighty-nine"?
- Why is 0.8 greater than 0.452, even though 452 is greater than 8?
- For a set of decimal numbers, is the longest number always the greatest number? Explain how you know.
- _____ is less than _____, because ...

- Understand and use place value for decimals, measures and integers of any size
- Order positive and negative integers, decimals and fractions; use the number line as a model for ordering of the real numbers; use the symbols =, ≠, <, >, ≤, ≥

Compare and order decimals



Teaching approaches

Show some numbers.

56.07

60.47

56.47

60.07

56.7

Discuss what is the same and what is different about the numbers.

Ask students to compare the numbers, justifying their comparisons. If necessary, encourage the use of a place value chart. Then ask them to write the numbers in ascending/descending order.

Repeat with other decimal numbers and integers, as appropriate.

 Measure the heights of five students to the nearest hundredth of a metre.

0.95 m

1.04 m

1.14 m

1.24 m

1.30 m

Ask students to compare and order the heights. Challenge them to write a new height that will be the second smallest in the list.

Key vocabulary

compare look at quantities to find out whether their

value is greater than, equal to or less than

digit any numeral from 0 to 9

leading digit first digit in a number

decimal number number with a whole and a fractional part,

for example 3.2, 5.43

place value value of a digit that relates to its position in

a number

- Students compare and order decimal numbers, such as those for the diameters of planets.
- Support curriculum Year 7 Autumn Block 4 Step 8 Compare and order decimals
- Students will later compare decimal numbers with fractions and percentages.
- Challenge students to compare and order decimal numbers that have different units, for example 95 g, 1.2 kg, 0.829 kg, 2 kg, 3521 g.

Round to powers of 10



Notes and guidance

In this small step, students round numbers to different degrees of accuracy.

Use number lines to help develop thinking about which two multiples a number is between and to compare a number to the midpoint of the number line. Emphasise "nearest" as meaning proximity, encouraging students to think about where a number is positioned on a number line, rather than rote-learned rules. Students need to understand the convention that numbers halfway between two multiples round to the greater multiple, for example 285 rounds to 290 to the nearest 10. Highlight that it is possible to get the same result when rounding a number to different degrees of accuracy. For example, 8004 rounds to 8000 to the nearest 10, the nearest 100 and the nearest 1000

Misconceptions and common errors

- Students may get confused if the term "round up/down" is used, for example rounding 3185 to 2000 to the nearest 1000
- Students may misapply the convention regarding numbers halfway between two multiples and not look at the correct place value column to make their decisions about rounding, for example rounding 635 to 700 to the nearest 100

Mathematical talk

- Which multiples of 10/100/1000/10000 does the number lie between?
- How can you represent the rounding of this number on a number line?
- Which place value column should you look at to round the number to the nearest 10/100/1000/10000?
- What do you do if a number lies exactly halfway between two multiples?
- What is the most appropriate way of rounding the number of people living in the UK? What is the most appropriate way of rounding the number of people at a concert or in your school?

	1 1 4 4 1	
	rounded to the nearest	15
_		'>

National Curriculum links

- Understand and use place value for decimals, measures and integers of any size
- Round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]

Round to powers of 10



Teaching approaches

 Ask students to draw a number line, labelling the start and end points with two numbers such as 340 and 350. Ask them to mark the position of 345

Give students some numbers and ask them to use their number line to identify which multiple of 10 each number is closer to, explaining the meaning of "round to the nearest 10".

Repeat with different start and end point values and other powers of 10

 Ask students to fill in a 3 by 3 grid with numbers between 0 and 1000

16	182	249
602	322	503
762	879	933

Play "rounding bingo" by asking students to, for example, cross out a number that rounds to 500 when rounded to the nearest 100

Key vocabulary

round replace a number with an approximate value

nearest closest in distance or time

power of 10 repeated multiplication by 10. For example,

100 (10^2 or 10×10) is the second power of 10

midpoint halfway between two intervals on a number line

or two numbers

convention agreed way that something is done, for example

how something is named or the way notation

is presented

- Support curriculum Year 7 Autumn Block 7 Step 3 Round numbers to the nearest 10, 100 and 1000
- Students will use the skill of rounding numbers later, when estimating answers to calculations.
- Challenge students to identify a sensible degree of accuracy for rounding, depending on the context of the given number.

Round to the nearest integer



Notes and guidance

Building on the previous step, in this small step students round numbers to the nearest integer.

Students need to be confident with identifying the previous and next integer of a number and finding the midpoints between those integers. Number lines can be used to help them identify which integer the number is closer to. They may need reminding that when a number is exactly halfway between two integers, the convention is to round to the greater integer. For example, 2.5 rounds to 3 to the nearest integer.

Misconceptions and common errors

- Students may focus on rules about "rounding up/down" instead of looking at the nearest integer, for example rounding 4.1 to 3
- Students may misapply the convention regarding numbers halfway between two integers and not look at the correct place value column to make their decisions about rounding, for example rounding 7.15 to 8 to the nearest integer.

Mathematical talk

- Which two integers does the number lie between?
- How can you represent the rounding of this number on a number line?
- How do you label halfway between the two integers on a number line?
- Which place value column should you look at to round the number to the nearest integer?
- What happens if a number lies exactly halfway between two integers?

lies between the integers and	
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_	rounded	to the	nearest	integer	is
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National Curriculum links

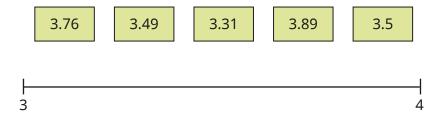
- Understand and use place value for decimals, measures and integers of any size
- Round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]

Round to the nearest integer



Teaching approaches

Display some numbers and a related number line.



Ask students to estimate the positions of the numbers on the number line and ask questions.

- Which numbers are closer to 3/4?
- What is each number rounded to the nearest integer?

Repeat with other sets of numbers.

 Show four digit cards and ask students to use each digit card once to make all the statements correct.



- ___.9 rounded to the nearest integer is 6
- 3. __ rounded to the nearest integer is 3
- __.45 rounded to the nearest integer is 7
- 9.3 rounded to the nearest integer is 9

Key vocabulary

round replace a number with an approximate value

nearest closest in distance or time

integer whole number that can be positive or negative

midpoint halfway between two intervals on a number line

or two numbers

convention agreed way that something is done, for example

how something is named or the way notation

is presented

- Support curriculum Year 8 Spring Block 4 Step 3 Round to the nearest integer
- Students will use the skill of rounding numbers later when estimating answers to calculations.

Round to decimal places



Notes and guidance

In this small step, students round numbers to a specified number of decimal places.

It is crucial for students to be able to identify multiples of 0.1, 0.01, and so on, to determine which two multiples a given number lies between and which multiple it is closest to.

As in the previous steps, number lines are a useful representation. Remind students that numbers exactly halfway between two values round to the greater value. For example, 2.35 rounds to 2.4 when rounded to 1 decimal place. Highlight the fact that integers, including zero, can be written as numbers with one or more decimal places, for example 7 = 7.0

Misconceptions and common errors

- Students may get confused if the term "round down/up" is used, for example rounding 8.34 to 8.2 when rounding to 1 decimal place.
- Students may omit the zero in their rounded answer, which can indicate the accuracy with which they have rounded.
 For example, when rounding 3.99 to 1 decimal place, they may write 4 instead of 4.0

Mathematical talk

- If rounding to _____ decimal place(s), which place value column do you need to look at to decide where to round to?
- Using the number line, which multiple of _____ is the number closer to?
- What do you do to round a number that is halfway between two multiples of ______?
- _____ lies between ____ and ____
- _____ is closer to _____ than ____
- rounded to _____ decimal place(s) is _____

National Curriculum links

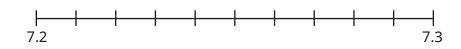
- Understand and use place value for decimals, measures and integers of any size
- Round numbers and measures to an appropriate degree of accuracy [for example, to a number of decimal places or significant figures]

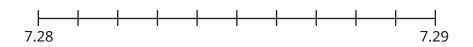
Round to decimal places



Teaching approaches

Show two number lines.





Ask students to discuss what is the same and what is different about the number lines.

Ask students to estimate the position of 7.281 on each number line and encourage them to use the number lines to round 7.281 to 1 decimal place, and then to 2 decimal places.

Repeat with other numbers and number lines, establishing how to round a decimal number to 1 or 2 decimal places. Encourage students to determine the correct multiples the number lies between for the degree of accuracy they are asked to round to. This can then be extended to rounding to 3 or more decimal places.

Key vocabulary

round replace a number with an approximate value

nearest closest in distance or time

decimal number number with a whole and a fractional part,

for example 3.2, 5.43

midpoint halfway between two intervals on a number

line or two numbers

- Students round a number from a calculator display to a given number of decimal places after solving scientific equations.
- Support curriculum Year 8 Spring Block 4 Step 4 Round to decimal places
- Students will later use rounding when estimating answers to calculations.



Powers of 10



Notes and guidance

In this extend step, students explore how to express numbers such as $10\,000$ as 10^4 , laying the groundwork for understanding standard form.

Calculators can be used to help introduce this concept and provide practice with large numbers, such as billions. This step builds on students' prior knowledge of powers from their learning of squares and cube numbers at Key Stage 2 and also place value, emphasising how the position and value of the digit "1" changes with different powers. Students practise converting between powers of 10 and standard numerical form, becoming proficient in representing numbers both ways.

Keep in mind, if students need more time to gain fluency with earlier steps, the content of this step will be covered again in Year 8

Misconceptions and common errors

- Students may think that they multiply the base by the power, for example $10^2 = 10 \times 2$
- Students may incorrectly add the same number of zeros after the 10 as the power of 10, for example writing 10³ as 10 followed by three zeros (10 000).

Mathematical talk

- What does "ten to the power of five" mean?
- What does the index tell you? How does this link to place value columns?
- How many times greater is 10⁵ than 100? Explain how you know.
- Why are very large numbers written as powers of 10?
- How can you write 1 000 000 as a power of 10?
- How can you use the power button on your calculator to check your answer?
- To work out 10 , I multiply 10 by itself _____ times.

National Curriculum links

 Use integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 and distinguish between exact representations of roots and their decimal approximations



Powers of 10



Teaching approaches

Show a completed powers of 10 table.

Power of 10	Calculation	Number
10 ²	10 × 10	100
10 ³	10 × 10 × 10	1000
104	10 × 10 × 10 × 10	10000

Ask students questions to develop understanding.

- What connections can you see?
- What calculation/power of 10/number comes next?
- How can you work out 10⁸?
- How can you write 1000000000 as a power of 10?

Check that students know how to use the power button on their calculators to check their answers.

• Show a calculation.

Discuss if there is a simpler way to write the calculation, encouraging students to start by writing each number as a power of 10

Key vocabulary

power way to express repeated multiplication of a

number using a base and an index

index small number to the right and above the base

number that shows how many times a number

or letter has been multiplied by itself

base number that is multiplied when using an index

power of 10 repeated multiplication by 10. For example,

100 (10^2 or 10×10) is the second power of 10

- Students use powers of 10 to perform order of magnitude calculations. They also use prefixes such as "giga-" and "mega-".
- Support curriculum Year 8 Spring Block 6 Step 2 Positive powers of 10
- Powers of 10 will be revisited in Year 8



Numbers greater than 1 in standard form



Notes and guidance

In this extend step, students learn how to write and interpret numbers such as 8×10^9 and 4.5×10^8

Students need to understand how standard form works, rather than just counting zeros. It is important to discuss the correct notation of $A \times 10^n$, where A is a number greater than or equal to 1 and less than 10, and n is an integer. Using non-examples such as 0.8×10^4 and $5 \times 10^{0.3}$ is a good way to deepen students' understanding. The intention in this block is for students to get a good understanding of the basics, rather than rushing to learning procedural rules. Standard form is studied in depth in Year 8

Misconceptions and common errors

- Students may write equivalent calculations that are not in standard form, for example 3200 as 32×10^2
- Students may misunderstand the convention 1 ≤ A < 10
 as representing numbers between 1 and 10, not including
 1 itself. For example, when writing 10 000 in standard form
 students may struggle to pick the correct value for A.
- Students may think that the power always corresponds to the number of zeros, for example writing 3400 as 3.4×10^2

Mathematical talk

- Why is standard form used?
- How can you convert numbers such as millions and billions easily into standard form?
- What is the same and what is different about how 3000 and 3500 are written in standard form?
- Does counting the zero digits in a number indicate the power of 10 that the number will have in standard form?
 Explain how you know.
- Why is 50×10^4 not correctly written in standard form?
- Why is $4 \times 10^{0.5}$ not correctly written in standard form?
- Give an example of where you might see a number written in standard form.
- The first number in standard form must be _____ than or _____ to 1 but _____ than 10

National Curriculum links

• Interpret and compare numbers in standard form $A \times 10^n$ 1 $\leq A < 10$, where n is a positive or negative integer or zero



Numbers greater than 1 in standard form



Teaching approaches

Display some numbers.

8000

9000

Model how to write each number as a multiplication and use powers of 10 to write the numbers in standard form.

$$8000 = 8 \times 1000$$

= 8×10^3

$$9000 = 9 \times 1000$$

= 9×10^3

Now display the number that lies halfway between 8000 and 9000

8500

Discuss how this number could be written in standard form before modelling the process of converting to standard form.

Repeat with other numbers such as 850, 8050 and 85000

- Show students some large numbers in context.
 - distance to the Sun ≈ 151 000 000 km
 - number of cells in the human body ≈ 36 000 000 000 000
 - number of stars in the Milky Way ≈ 100 000 000 000

Discuss why it is useful to write these numbers in standard form rather than as ordinary numbers.

Key vocabulary

power way to express repeated multiplication of a

number using a base and an index

index small number to the right and above the base

number that shows how many times a number

or letter has been multiplied by itself

standard form number that is written in the form $A \times 10^n$,

where $1 \le A < 10$ and n is an integer

power of 10 repeated multiplication by 10. For example,

100 (10^2 or 10×10) is the second power of 10

- Students write integers in standard form to represent quantities such as the speed of light.
- Support curriculum Year 8 Spring Block 6 Step 4 –
 Numbers greater than 1 in standard form
- Numbers greater than 1 in standard form will be revisited in Year 8



Negative powers of 10



Notes and guidance

In this extend step, students explore how to express numbers between zero and one as a power of 10

Students investigate decreasing powers of 10 and explore what happens when they reach powers of zero and below. They develop confidence with a negative power being equivalent to repeated division or multiplication by the reciprocal. For example,

$$10^{-2} = 1 \div 10 \div 10$$
 or $10^{-2} = 1 \times \frac{1}{10^2} = 1 \times \frac{1}{100}$

Students will have been introduced to negative numbers in Key Stage 2, but it may be useful to reinforce that, for example, -2 is greater than -4

Misconceptions and common errors

- Students may think that 10° = 0, instead of 1
- Students may misunderstand that a negative power is equivalent to repeated division, but instead believe that it is equivalent to multiplying by a negative value, for example $10^{-2} = -100$

Mathematical talk

- What is the difference between positive and negative powers of 10?
- What does "ten to the power of negative two" mean?
- What does the index tell you?
- Why is 10⁰ not equal to zero?
- Is 10^{-4} greater than or less than 10^{-2} ?
- Why are very small numbers written as powers of 10 rather than ordinary numbers?
- How can you write 0.000001 as a power of 10?
- How can you use the power button on your calculator to check your answer?

National Curriculum links

• Interpret and compare numbers in standard form $A \times 10^n$ 1 $\leq A < 10$, where n is a positive or negative integer or zero



Negative powers of 10



Teaching approaches

Show a partially completed table of powers of 10

Power of 10	Calculation	Answer
104	10 × 10 × 10 × 10	10 000
10 ³	10 × 10 × 10	
10 ²		
10 ¹		
10 ⁻¹	1 ÷ 10	0.1
10 ⁻²	1 ÷ 10 ÷ 10	
10 ⁻³		
10 ⁻⁴		
10 ⁻⁵		

Ask students questions to help them complete the table.

- What do you notice? What patterns can you see?
- How can you work out 10°?
- Ask students to enter the number 10 on a calculator.

Next ask them to divide by 10 and note the answer. Then to divide by 10 again and note the answer, and keep going.

Ask what happens after several divisions and why they think this is the case, highlighting why it is more efficient to write very small numbers as a power of 10

Key vocabulary

power way to express repeated multiplication of a

number using a base and an index

index small number to the right and above the

base number that shows how many times a number or letter has been multiplied by itself

negative number number that is less than zero

power of 10 repeated multiplication by 10. For example,

100 (10^2 or 10×10) is the second power of 10

- Students use powers of 10 to perform order of magnitude calculations. They also use prefixes such as "milli-" and "nano-".
- Support Year 9 Autumn Block 7 Step 2 Negative powers of 10
- Negative powers of 10 will be revisited in Year 8
- Challenge students to consider the different ways a number can be written using powers of 10, for example writing 0.00001 as 10^{-5} or $10^{-2} \times 10^{-3}$



Numbers between 0 and 1 in standard form



Notes and guidance

In this extend step, students learn how to write and interpret numbers such as 2×10^{-3} and 2.4×10^{-3}

Students can explore the patterns and connections between decimal numbers and standard form. It is important to again discuss the correct notation of $A \times 10^n$, where A is a number greater than or equal to 1 and less than 10, and n is an integer. Encourage discussions about why standard form is used and how it can be more efficient to write very small numbers in this way. This learning will be revisited in Year 8

Misconceptions and common errors

- Students may write an equivalent calculation that is not in standard form, for example $0.0047 = 0.47 \times 10^{-2}$
- Students may count decimal place "jumps" instead of the power of 10 that a value of A is divided by.
- Students may ignore later digits in a number, for example in 0.05001, using 5 for the value of A, rather than 5.001
- Students may misinterpret negative powers, for example $3 \times 10^{-2} = -0.03$

Mathematical talk

- What is the same and what is different about writing large numbers and small numbers in standard form?
- Where might you see and use standard form?
- Why is it useful to write numbers such as 0.0000000091 in standard form?
- What is the same and what is different about 3.5×10^4 and 3.5×10^{-4} ?
- Explain why 4×10^{-2} is greater than 4×10^{-3}
- Is 58×10^{-4} written in standard form? Explain how you know.
- Are negative powers of 10 always negative numbers?
 Explain how you know.
- The first number in standard form must be _____ than or equal to 1 but _____ than 10

National Curriculum links

• Interpret and compare numbers in standard form $A \times 10^n$ 1 $\leq A < 10$, where n is a positive or negative integer or zero



Numbers between 0 and 1 in standard form



Teaching approaches

- Give some examples of small numbers in context.
 - diameter of a red blood cell ≈ 0.0000075 m
 - width of an atom ≈ 0.0000000001 m

Discuss why it is useful to have a more efficient method for writing small numbers.

 Show students the partially completed table and ask them to complete it.

Number	Calculation	Standard form
0.5	5 × 0.1	5 × 10 ⁻¹
0.05	5 × 0.01	5 × 10 ⁻²
0.005		
0.0005		

Ask students to discuss any patterns they can see, encouraging them to use their knowledge of negative powers of 10 from the previous step to make the connection that 0.1 is equivalent to 10^{-1} . Encourage them to use calculators to check answers.

Key vocabulary

power way to express repeated multiplication of a

number using a base and an index

index small number to the right and above the base

number that shows how many times a number

or letter has been multiplied by itself

standard form number that is written in the form $A \times 10^n$,

where $1 \le A < 10$ and n is an integer

power of 10 repeated multiplication by 10. For example,

100 (10^2 or 10×10) is the second power of 10

- Students write numbers in standard form when considering the size of various biological structures.
- Support curriculum Year 9 Autumn Block 7 Step 3 –
 Numbers between 0 and 1 in standard form
- Numbers between 0 and 1 in standard form will be revisited in Year 8
- Challenge students to write numbers such as 48×10^{-3} and 0.3×10^{-7} correctly in standard form.